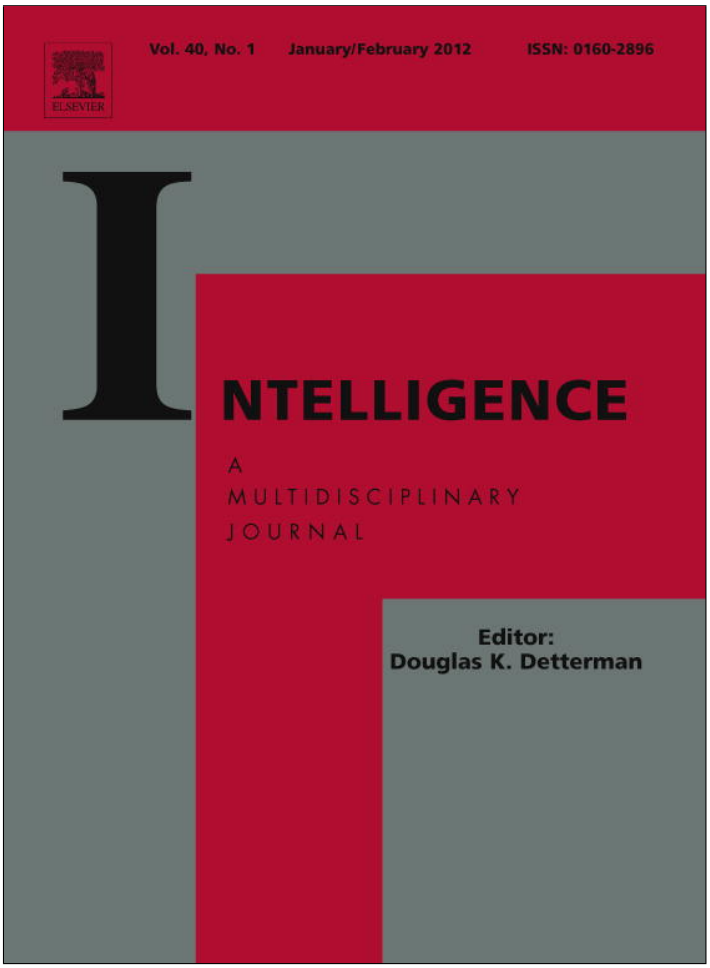
Provided for non-commercial research and education use.
Not for reproduction, distribution or commercial use.




This article appeared in a journal published by Elsevier. The attached copy is furnished to the author for internal non-commercial research and education use, including for instruction at the authors institution and sharing with colleagues.

Other uses, including reproduction and distribution, or selling or licensing copies, or posting to personal, institutional or third party websites are prohibited.

In most cases authors are permitted to post their version of the article (e.g. in Word or Tex form) to their personal website or institutional repository. Authors requiring further information regarding Elsevier's archiving and manuscript policies are encouraged to visit:

http://www.elsevier.com/copyright

Author's personal copy

Intelligence 40 (2012) 23–32

Contents lists available at SciVerse ScienceDirect

Intelligence

# Can fast and slow intelligence be differentiated?

Ivailo Partchev [a,c], Paul De Boeck [b,c,*]

[a] Cito, Amsterdamsesteenweg 13, 6814CM Arnhem, The Netherlands
[b] Department of Psychology, University of Amsterdam, Roeterstraat 15, 1018WB, Amsterdam, The Netherlands
[c] Department of Psychology, K.U. Leuven, Tiensestraat 102, B-3000 Leuven, Belgium

ARTICLE INFO

*Article history:*
Received 24 August 2011
Received in revised form 18 October 2011
Accepted 10 November 2011
Available online 6 December 2011

*Keywords:*
Speed
Power
Abilities
Intelligence
IRT

ABSTRACT

Responses to items from an intelligence test may be fast or slow. The research issue dealt with in this paper is whether the intelligence involved in fast correct responses differs in nature from the intelligence involved in slow correct responses. There are two questions related to this issue: 1. Are the processes involved different? 2. Are the abilities involved different? An answer to these questions is provided making use of data from a Raven-like matrices test and a verbal analogies test, and the use of a psychometric branching model. The branching model is based on three latent traits: speed, fast accuracy and slow accuracy, and item parameters corresponding to each of these. The pattern of item difficulties is used to draw conclusions on the cognitive processes involved. The results are as follows: 1. The processes involved in fast and slow responses can be differentiated, as can be derived from qualitative differences in the patterns of item difficulty, and fast responses lead to a larger differentiation between items than slow responses do. 2. The abilities underlying fast and slow responses can also be differentiated, and fast responses allow for a better differentiation between the respondents.

© 2011 Elsevier Inc. All rights reserved.

## 1. Introduction

It is still an issue to what extent intelligence tests measure mental speed and to what extent they measure mental capacity and how much these two are related. The issue of speed and power is an old one (Kelley, 1927) but seems still unresolved when speed and power refer to factors internal to the test (Anastasi, 1976; Dennis & Evans, 1996; Gulliksen, 1950; van der Linden, 2009). A popular present-day approach is to look for pure measures of speed and power *external to the test*. This has led to the elementary cognitive task (ECT) approach (e.g. Neubauer & Bucik, 1996; Sheppard & Vernon, 2007). These tasks are different from the tasks in an intelligence test but they are assumed to tap basic features of the cognitive system that underlie the level of performance in the common and more complex type of intelligence test. The logic behind this approach is what Hunt (1978) has described as the cognitive correlates method. For the measurement of mental speed or speed of information processing, one makes use of simple response time tasks such as the Hick (1952) paradigm and inspection time tasks (Vickers, Nettelbeck, & Willson, 1972). For the measurement of mental capacity or mental power, working memory capacity tasks are used, such as the n-back memory span task (Baddeley, 1986). An interesting alternative approach can be found in temporal and non-temporal discrimination tasks (Troche & Rammsayer, 2009a; 2009b), respectively for speed and power.

Sheppard and Vernon (2007) and Grudnik and Kranzler (2001) provide clear evidence for a positive correlation between intelligence scores and speed of information processing, and Gray, Chabris, and Braver (2003) for a correlation with working memory capacity. In a recent study Waiter et al. (2009) have correlated both with scores on the Raven's Matrices. Moderately high positive correlations were found for both, but these correlations could not be explained

* Corresponding author at: Department of Psychology FMG, University of Amsterdam, Weesperplein 4, 1018XA Amsterdam, The Netherlands.
*E-mail address:* paul.deboeck@uva.nl (P. De Boeck).

0160-2896/$ – see front matter © 2011 Elsevier Inc. All rights reserved.
doi:10.1016/j.intell.2011.11.002

Author's personal copy



through the mediation of brain activation as measured in their study. One may readily conclude that mental speed in the form of speed of information processing and power in the form of working memory capacity are two important components, but that it is unclear thus far through which kind of brain activity they fulfill that role.

Early on in the history of psychology, it was assumed, for example, by Spearman (1927) that speed and power rely on one and the same ability. A theoretically interesting explanation for such a position is found in Vernon's (1983) theory on the relationship between speed of processing and working memory. The theory implies that fast processing is a way to deal with decay in the available information. The elements one has to operate with may have vanished by the time a slow processing mind is done, so that the cognitive task remains unresolved or needs a whole new trial. Indirect empirical support for the theory is provided, among others, by Vernon, Nador, and Kantor (1985) and Vernon and Kantor (1986). The implication is that a higher speed of information processing provides the brain with a larger working memory capacity. On the other hand, there is empirical evidence that mental speed is differentiated from working memory capacity (Rypma & Prabhakaran, 2009; Troche & Rammsayer, 2009a; Waiter et al., 2009; Wilhelm & Schulze, 2002). For example, Rypma and Prabhakaran (2009) come to the interesting conclusion that neural efficiency based on direct connectivity instead of executive control processes can compensate for limited working memory capacity, which is a somewhat different formulation than Vernon's (1983) because it implies the independent existence of working memory limitations. However, both theories imply that slow responses are of a different nature than fast responses. Fast responses are based on more automatic direct-link mediated processing while slow responses are based on repeating one's cognitive work and/or more controlled processing. The difference between automatic and controlled is described in a seminal paper by Shiffrin and Schneider, (1977).

Recently, there seems to be a renewed interest in an approach *internal to the test* for the study of speed and capacity (level of performance), without relating test results to elementary external tasks (Davison, Semmes, Huang, & Close, 2011; Partchev et al., in press; Semmes, Davison, & Close, 2011). Internal to the test means that response time and accuracy of the item responses from the test are considered the basic data. However, these studies focus on the measurement and study of speed and capacity rather than on two possibly different capacities, one referring to the accuracy of fast responses and the other to the accuracy of slow responses.

### 1.1. Fast and slow intelligence

A common sense formulation of the possible automatic vs. controlled processing difference between fast and slow would be that respondents start with relying on acquired knowledge and familiar strategies, and that they shift to reasoning instead of knowledge or to less familiar strategies and ad hoc constructed strategies. There may be also other differences than between automatic and controlled that can differentiate between fast and slow and are not necessarily contradictory to the difference between automatic and controlled. For example, if multiple strategies are available to solve a problem and when these strategies differ in how much time they take, then one may first try the fastest strategy, and, when it is not successful switch to a slower strategy. Another example is switching from a forward strategy, such as reasoning from the problem to the correct solution, to a reverse strategy, starting from possible responses, in the hope one can eliminate all but one (in a multiple-choice task).

The suggestion that slow responses are based on a different kind of cognitive processing compared to fast responses, raises the issue whether fast and slow accuracy rely on different abilities. This means that there are two distinct aspects involved. First, is the processing different? Second, are the abilities different? Different processes and strategies can indicate different abilities, but may alternatively rely on the same ability. In the following it is explained how these two aspects can be investigated in a disentangled way.

If, when fast and slow responses are compared, the corresponding item difficulties appear to differ in more than their level (a quantitative difference), then the items in question must have been solved in a different way depending on whether the responses are fast or slow. The definition of "item difficulty" will be based on the item response model (IRT model) that will be used. The model allows for two sets of item difficulties: one for fast responses and another for slow responses. Differences between fast and slow that go beyond differences in overall level of difficulty imply more than just going faster or slower through the same processes. Therefore, a qualitative difference in processing can be inferred from a difference in the pattern of item difficulties. The advantage of this strategy is that no additional observations on the way of processing are needed that may interfere with the spontaneous way of processing. However, the if/then is not necessarily symmetrical: different ways of processing may still lead to the same pattern of difficulties derived for a given set of respondents and a given test.

For example, let us consider two strategies for addition items with numbers up to 99. Both strategies consist of adding the units first and then the tens. For example, when presented with the addition "37 + 46", the two derived elementary additions are "7 + 6 = 13" and "3 + 4 = 7". The difference between the strategies lies in how these results are processed. Following the first strategy, neither the 13, nor the 7 are written down. Only the 3 of the 13 is written down, while the 1 is held in mind in order to be added to the 7, leading to 83. Following the second strategy, both intermediate addition results are written down as 7°13, and the 1 is moved forward: 7 + 1°3, leading again to 83. The two strategies do not differ when the sum of the units is smaller than 10, and therefore, the pattern of difficulties would not differ either. When the sum is 10 or larger indeed, one may expect that the difference between carry-over additions and additions without will be larger using the first strategy than when using the second strategy. However, when it comes to response times, the first strategy has an advantage, because it does not require writing down the intermediate result (7°13) and its transformation into the final result (7 + 1°3). This example shows also how it is possible that slower responses rely on different processes than fast responses.

While qualitative differences in processing can be inferred from the item difficulties, qualitative differences in ability can be inferred from the person latent traits. The definition of a

Author's personal copy



latent trait will be based on the item response model (IRT model) that will be used. The model allows for two different latent traits: one for fast responses and another for slow responses. A qualitative difference in ability can be inferred from the relationship between the two latent traits (fast and slow) across persons. If, when fast and slow responses are compared, the corresponding latent traits appear to differ in more than their level (a quantitative difference), different abilities must be involved. However, also this if/then is not necessarily symmetrical: qualitatively different abilities may still show as only one for a given set of respondents and a given test. For example, if the items of a test are (perhaps accidentally) selected to lie approximately on one straight line in a two-dimensional factor space, then the test would look as one-dimensional, while in fact two qualitatively different abilities are involved. In a similar way also the set of respondents may be of a kind that two factors are almost perfectly correlated.

In sum, qualitative process differences can be inferred from the across-item pattern of difficulties, and qualitative ability differences from the across-person pattern of the latent trait values. There are four possible outcomes comparing fast and slow responses:

1. no qualitative differences for items, and neither for persons: no reason to conclude that the processes or abilities are different;
2. no qualitative differences for items, but qualitative differences for persons indeed: the abilities are different but they do not have empirical consequences for the item difficulties;
3. qualitative differences for items, but not for persons: the processes are different but they do not lead to an empirical differentiation in terms of abilities;
4. qualitative differences for items and for persons: the processes are different and the corresponding abilities are different as well.

In order to explain the logic, let us look at the third possibility, where the pattern of item difficulties differs in a qualitative way while the abilities cannot be differentiated. A possible example comes from the domain of athletic sports. Sprint and far jump imply different processes (running and jumping), but both rely on one's speed. The performance in both is highly correlated. In terms of the topic under study, a possible conclusion is that fast and slow accuracy are based on two different kinds of processes that are nevertheless rooted in the same ability or in two (almost) perfectly correlated abilities. That the processing is different is not necessarily a problem for the measurement of intelligence as an underlying ability. However, if fast and slow accuracy measure different abilities, then the meaning of an intelligence score will depend on the proportion of fast and slow responses the score is based on. Individual differences in these proportions lead to measurement problems. It is therefore an important issue whether fast intelligence and slow intelligence are the same or not. As far as we know this issue has not been investigated before. It is different from the issue whether response times are correlated with accuracy scores and it is also different from the issue how much speeded and non-speeded tests are correlated (Davison et al., 2011; Semmes et al., 2011). However, the latter is not unrelated to the issue we are focusing on, because time pressure may induce the kind of processing at the basis of fast responses and prevent a way of processing that is used for slow responses. Practically speaking, a speeded test may favor the measurement of fast intelligence, while a (relatively) non-speeded test would measure a mix of both. It would therefore be of interest if the two could be disentangled. Theoretically speaking, if fast and slow intelligence were different, then intelligence needs to be conceptualized as a toolbox of diverse abilities respondents have available for solving a given kind of problem.

### 1.2. Aim of the study

The aim of the study is twofold and refers to time-homogeneity vs. time-heterogeneity of responses to items from an intelligence test. The concept of time-homogeneity applies to the underlying processes *and* to the underlying abilities. Does a larger response time mean that more of the same processes are executed or equivalently, that they are executed for a longer time, or do larger response times imply a different kind of processing? In a similar way, is the ability the same independent of the response time, or do slower responses rely on a different ability? Although there is much to say in favor of the time-homogeneity assumption, we have given reasons for the possibility of time-heterogeneous processing. When the processes are different, it does not necessarily follow but neither would it surprise that also the abilities are different.

Two kinds of inductive tasks will be investigated: verbal analogies and a test with matrices, and of each kind, two sets of items will be investigated. In this way it can be checked whether the results generalize within the same domain of tasks and between different domains of tasks. Inductive reasoning is a basic kind of intelligence and it is even considered to tap g in the first place (Carroll, 1993; Gustafsson, 1984; Kvist & Gustafsson, 2008).

### 1.3. Distinguishing between fast and slow responses

In order to practically investigate the issue of fast versus slow intelligence, an operational definition is needed of what is fast and what is slow. By definition, speed is gradual, so that the optimal approach is to make gradual distinctions. However, as will become clear from the following, this would require a new kind of model for the data and would therefore be a topic of research in itself. There are of course already models for response time and also for response time in combination with accuracy (e.g., van Breukelen, 2005, van der Linden, 2009; Wang & Hanson, 2005), but they are not models with a gradual change of the nature of the processing and the ability involved with the gradual elapse of time.

In order to make use of existing models for our research issue, we will work here with a categorical definition of speed. In fact two definitions will be used, so that it can be checked whether the results are not method specific. The first definition is a within-person definition, based on an intra-individual median split. A fast response is a response that belongs to the fastest half of responses of the person in question. For each person, a fast and a slow subset of items is determined. The second definition is a within-item definition, based on an item-wise inter-individual median split. A fast response is a response that belongs to the fastest half of responses to the item

Author's personal copy



in question. For each item, a fast and a slow subset of persons is determined.

The within-person split makes all respondents about equal with respect to speed (all persons have as many fast as slow items), while it respects the full range of item differences. A possible drawback would be a too large divergence between the fast and slow item subsets, when too many items end up either only in the fast subset or in the slow subset. It will therefore be checked to what extent each item populates the two kinds of item subsets. The within-item split makes all items about equal with respect to speed (all items have as many fast as slow persons), while it preserves the full range of individual differences in speed. A possible drawback would be a too large divergence between the fast and slow person subsets, when too many persons end up either only in the fast or in the slow subset. It will therefore be checked to what extent each person populates the two kinds of person subsets.

## 2. Method

### 2.1. Model

The model that will be used is a two-level branching model. It is a nested version of the sequential continuation ratio model formulated by Tutz (1990), but it can be considered also an individual-differences version (Smith & Batchelder, 2008; Klauer, 2010) of the binomial tree process models described by Batchelder and Riefer (1999) and Erdfelder et al. (2009). The branching structure is shown in Fig. 1. The highest branching level differentiates between fast and slow. Within the fast and the slow branch, a further differentiation is made between correct and incorrect. This results in three nodes and four categories of responses, the leaves of the branching tree. The probability of the two branches from the same node depends on the item and on the respondent. The model specifies the probabilities as a logistic function of the item difficulty and the person ability:

$$\pi_{pis} = \exp\left(\eta_{pis}\right)/\left(1+\exp\left(\eta_{pis}\right)\right) \tag{1}$$

where $\pi_{pis}$ is the probability of person $p$ ($p=1,..,P$) working on item $i$ ($i=1,..,I$) to go left at node $s$ ($s=1,..,S$);where

$$\eta_{pis} = \theta_{ps} - \beta_{is} \tag{2}$$

so that $\eta_{pis} = \log(\pi_{pis}/(1-\pi_{pis}))$,where $\theta_{ps}$ is the ability of person $p$ that makes him go left at node $s$, with a multivariate distribution: $\boldsymbol{\theta}_p \sim MVN(\mathbf{0}, \Sigma_\theta)$, which implies $S$ variance parameters and $S\times(S-1)/2$ covariance or correlation parameters; where $\beta_{is}$ is the difficulty of item $i$ which makes it more difficult for a respondent to go left at node $s$.

In the present application node 1 is for fast vs. slow, node 2 is for correct vs. incorrect following the fast branch, and node 3 is for correct vs. incorrect following the slow branch. This means that the probabilities $\pi_{pi2}$ and $\pi_{pi3}$ are in fact conditional probabilities: probabilities of a correct response given that the response is fast or slow, respectively.

The branching model implies that a response can be recoded into three binary sub-responses as indicated in Table 1, while each time only two of the three are observed. If the first sub-response is fast, the third sub-response is missing. If the first sub-response is slow, the second sub-response is missing. It means that whether a sub-response is missing depends on another sub-response observation.

The full model follows from the probabilities given in Table 1 and Eq. 1. An assumption of the model is that the sub-responses are independent, conditional on the latent traits $\theta_{p1}$ to $\theta_{p3}$. This may seem a questionable assumption, but note that the model does allow for dependence on the latent level, such as a correlation between latent speed and latent accuracy. Dependence on the level of the latent variables is not a problem for the model. Another potential problem is the missingness of sub-responses 2 and 3 depending on sub-response 1. Whether a sub-response is missing is not completely at random (MCAR), but, because the missingness depends solely on the first sub-response (and not also on a latent variable), we are dealing with data "missing at random" (MAR). Just as MCAR, also MAR is not a problem for a maximum likelihood based model estimation.

In line with the common practice in item response theory and structural equation modeling, the abilities are defined as

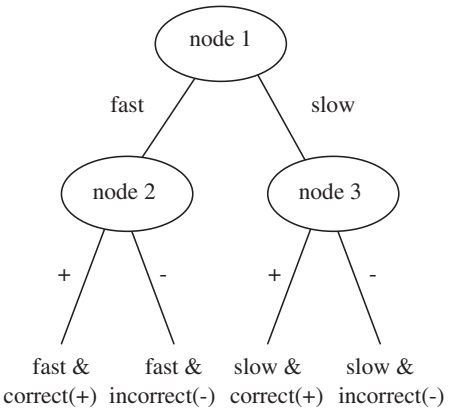



**Fig. 1.** Branching structure at the basis of the model to disentangle fast and slow intelligence.

Author's personal copy



**Table 1**
The branching model for fast and slow intelligence.

| Responses | Sub-responses | | | Probability |
|---|---|---|---|---|
| | $s=1$ | $s=2$ | $s=3$ | |
| 1 Fast and correct | 1 | 1 | – | $\pi_{pi1}\pi_{pi2}$ |
| 2 Fast and incorrect | 1 | 0 | – | $\pi_{pi1}(1-\pi_{pi2})$ |
| 3 Slow and correct | 0 | – | 1 | $(1-\pi_{pi1})\pi_{pi3}$ |
| 4 Slow and incorrect | 0 | – | 0 | $(1-\pi_{pi1})(1-\pi_{pi3})$ |

random variables with a covariance structure. The number of parameters for the abilities is therefore $S(S+1)/2$: $S$ variances and $S(S-1)/2$ covariances. In line with the same common practice, the item difficulties are defined as fixed effects. The number of parameters for the item difficulties is therefore $I\times S$. The variances and correlations of the $S$ sets of difficulties are not model parameters. The branching model will be estimated with ConQuest Version 2.0 (Wu, Adams, Wilson, & Haldane, 2007), but other IRT software allowing for missing data can be used for the same purpose.

### 2.2. Hypothesis testing

The model as in Eq. 1 and Table 1 will be estimated also in constrained versions in order to test the hypothesis that fast and slow processing and the corresponding abilities can be differentiated. Let us call the general model 3P&3I. It contains three abilities and three sets of difficulties. The first constrained model is the 2P&3I model, and it differs from the general model in that there is only one ability for fast and slow responses ($\theta_2=\theta_3$). The second constrained model is the 3P&2I model, and it differs from the general model in that there is only one set of difficulties for fast and slow responses ($\beta_2=\beta_3$). Finally, the third constrained model is the 2P&2I model with only one ability and one set of difficulties for slow and fast responses.

The hypothesis will be tested comparing models in three ways. First, the 3P&3I model will be compared with the 2P&3I, 3P&2I, and 2P&2I models, on the basis of the information criteria AIC (Akaike, 1974) and BIC (Schwarz, 1978). Second, the 3P&3I model will be compared with the 3P&2I model with the regular likelihood-ratio test to compare nested models, in order to test whether beside item difficulties for speed, two sets of item difficulties are needed instead of one for both fast and slow responses. Third, the 3P&3I model will be compared with the 2P&3I model with a mixture $\chi^2$-test for random effects (Molenberghs & Verbeke, 2003), testing three versus two random effects. The three random effects refer to latent speed and the fast and slow accuracy abilities, whereas two means that fast and slow accuracy rely on the same ability. The reason for the difference between testing 3P&3I versus 3P&2I and 3P&3I versus 2P&3I is that items are modeled with fixed effects and persons with random effects. The regular likelihood ratio test is not valid for random effects because the null hypothesis of zero variance is located on the boundary of the parameter space.

Using the data derived from the within-item split, it was not possible to estimate the item difficulties for speed, because the within-item split has made the items practically equal with regard to speed. This means that for the within-item split data, in practice a 3I model is a 2I model (only difficulties for fast and slow accuracy, but not for speed) and a 2I model is a 1I model (one set of item difficulties, for both fast and slow response). A similar problem did not occur for the abilities and the within-person split because abilities are random effects, but the variance estimate for the latent speed variable was extremely small as may be expected.

### 2.3. Intelligence tests

The first test is a *verbal analogies test*. Verbal analogies have attracted much attention in the study of intelligence (Spearman, 1927; Sternberg, 1977; Whitely, 1976). They are the most commonly used type of analogy items in intelligence tests (Ullstadius, Carlstedt, & Gustafsson, 2008), and they are used also for the Scholastic Aptitude Test (SAT) and Graduate Record Examination (GRE). Verbal analogy tests measure g and to some extent also crystallized intelligence (Bejar, Chaffin, & Embretson, 1991; Levine, 1950; Thurstone, 1938), the latter depending on how much word knowledge is required (Ullstadius et al., 2008). In sum, verbal analogies may be considered a very popular type of intelligence test, and a measure of core aspects of intelligence. We use data from the calibration studies for a computerized adaptive test with multiple-choice items developed by Hornke (Hornke & Rettig, 1993; Hornke, 1999; Hornke, 2001). The tests for the calibration study were administered in a computerized but non-adaptive format, with a very generous time allowance of 180 s per item (Hornke & Wilding, 1997).

The second test is a *Raven-like matrices test* (Hornke & Habon, 1986; Hornke & Wilding, 1997). Matrices are a type of inductive reasoning task supposed to tap on inductive reasoning (Carroll, 1993; Marshalek, Lohman, & Snow, 1983; Schweizer, Goldhammer, Rauch, & Moosburger, 2007), although in Gustafsson's (1984) well-known analysis (1984) matrices define a first-order factor Cognition of Figural Relations (CFR) which is distinct from Induction (I). On the second level, both CFR and I loaded on Fluid Intelligence, which in turn had a 1.00 loading on g as a third-order factor. Just as verbal analogies have a secondary loading, also matrix tests sometimes have modest loadings on spatial ability (e.g., Schweizer et al., 2007). Also matrix tests may be considered a very popular type of intelligence test, and a measure of core aspects of intelligence. They share with verbal analogy tests a focus on inductive reasoning and g, while they are perhaps not really pure tests either. We do not consider this a disadvantage, it may contribute to the generalizability of the findings. The specific test is based on an item design developed by Hornke and Habon (1986) and described also by Hornke (2001). Three types of design factors are used: type of rules, number of rules, and perceptual organization of the elements. A multiple-choice response format is used for the items. A full description of the items can be found in Hornke and Habon (1986). The test was administered in a computerized but non-adaptive format, with a very generous time allowance of 180 s per item (Hornke & Wilding, 1997).

Author's personal copy



### 2.4. Data sets

Two data sets will be analyzed. Both are subsets of a different very large data set, one on verbal analogies with 25 forms of 24 items each and a total N of more than 12,000, and one on matrices with 76 forms of 12 items each and a total N of about 30,000. The subsets are selected to be comparable, so that the findings regarding verbal analogies can be compared with the findings regarding matrices. All respondents were at age between 17 and 27 as of the time of testing, with a median age of 20 (Quartile 1 = 19, Quartile 3 = 21). Almost all were male. With respect to schooling level and occupation, they represented a realistic sample of the male population in this age group.

For the *verbal analogies data* the first four items of each form are a stub common to all 25 forms, and the corresponding responses were discarded. The remaining 20 items in each form overlap by the same ten items. The first five forms (60 items) were selected, and from this set of items, 36 are selected in a random way. On the person side every fifth examinee was selected, so that the total N is 726. Two items of the 36 had to be dropped because of extreme skewness: at least one of the four cells in the cross-tabulation of correct vs. incorrect by slow vs. fast had a zero count, which left us with 34 items and a dataset with responses missing by design. The block design is such that there is a common overlap of 10 items per subgroup of respondents.

For the *matrices data*, again the first five forms were selected, a total of 36 items, as the forms overlap by the same six items. On the person side, every sixth examinee was selected, so that the total N is 503. Again one item had to be dropped for the same reasons as was the case for the verbal analogies, which left us with 35 items and a dataset with responses missing by design. The block design is such that there is a common overlap of six items per subgroup of respondents.

The two data sets have a comparable size and are a reasonable sample from the original huge data sets. However, in order to investigate the replicability of the results, two quite different subsets of the two original data sets are used with the same procedure, and the results are very similar.

## 3. Results

### 3.1. Description of the data

For the verbal analogies, the proportions of success range from 0.026 to 0.985, the mean response time is 17.97 s, and the standard deviation is 15.69. When fast vs. slow is defined on the basis of a within-person split, the minimum number of observations of fast or slow responses per item is 14. It means that, when the items are classified per respondent into a fast-response category and a slow-response category, none of the items is solely classified in one of these two categories. The minimum frequency of 14 for one item is rapidly increasing for the other items: 18, 37, 44, etc. When fast vs. slow is defined on the basis of within-item split, the minimum number of observations of fast or slow responses per persons is 0, and this frequency is observed for 35 respondents. This means that there are 35 respondents who always give either fast responses or slow responses. This minimum frequency does not increase fast for the other persons: it is 1 for 42 respondents, 2 for 59, 3 for 61, etc.

For the matrices, the proportions of success range from 0.102 to 0.772, the mean response time is 68.95 s., and the standard deviation is 51.46. When fast vs. slow is defined on the basis of a within-person split, the minimum number of observations of fast or slow responses per item is 20. It means that for none of the items, all respondents always give either a fast or a slow response. This minimum frequency of 20 for one item does not increase as rapidly as for verbal analogies: 21, 30, 31, 32, 38, etc. When fast vs. slow is defined on the basis of a within-item split, the minimum number of observations of fast or slow responses per respondent is 0, and this frequency is observed for 23 respondents. This means that 23 respondents are always giving either fast responses or slow responses. This minimum frequency does not increase fast for the other persons: it is 1 for 43 respondents, 2 for 42, 3 for 36, etc.

The difference between the two kinds of median split for the two tests can be explained by the much higher number of respondents than items. The chances of a zero frequency are much lower if the number of splits equals the number of persons instead of the number of items. The difference can also be an indication of the inter-respondent correlation across items being lower than the inter-item correlation across respondents. In other words, persons may differ more in their response times than items do. However, for both kinds of median split, the total data sets are sufficiently informative to estimate the models.

For the verbal analogies, the Cronbach alphas for the fast and slow responses are 0.746 and 0.705, respectively when the within-person split is used, and 0.701 and 0.643 when the within-item split is used. The corresponding coefficients for the matrices are 0.727 and 0.679, and 0.768 and 0.630, respectively. For both tests, the fast responses seem more somewhat more reliable than the slow responses.

### 3.2. Model comparison

Table 2 shows the goodness of fit results of the branching models. The results indicate that in all four cases the constrained models (2P&3I, 3P&2I, 2P&2I) are rejected against the unconstrained model (3P&3I) when a statistical test is used (a likelihood ratio test or a mixture $\chi^2$-test). Furthermore, the item fit statistics provided in the ConQuest output have p-values which are not of the kind to question the goodness of fit of the 3P&3I models. Using the weighted statistic, the p-value is always larger than 0.05, while using the unweighted statistic, the p-value is smaller than 0.05 only in 1 out of 70 cases (within-item split) and 1 out of 105 cases (within-person split) for the matrix items, and in 5 out of 68 cases (within-item split) and 5 out of 102 cases (within-person split) for the verbal analogy items. In other words, the goodness-of-fit test is never significant at 0.05 for the weighted statistic, and significant for about (or less than) 5% of the items for the unweighted statistic.

It may be concluded from these results that fast and slow intelligence seem to be differentiated with respect to the corresponding latent ability as well as with respect to the item difficulties. The same conclusion must be drawn relying on the AIC. However, following the BIC, the 3P&3I model seems

Author's personal copy



**Table 2**
Goodness of fit of the branching models for verbal analogies and matrices.

| Kind of median split | # parameters | Deviance[a] | AIC[b] | BIC[b] |
|---|---|---|---|---|
| Within persons | | | | |
| Verbal analogies | | | | |
| 3P&3I | 108 | 16,574 | *16,790* | 17,246 |
| 2P&3I | 105 | 16,612*** | 16,822 | 17,266 |
| 3P&2I | 74 | 16,783*** | 16,931 | *17,243* |
| 2P&2I | 71 | 16,821 | 16,963 | 17,262 |
| Within items | | | | |
| Verbal analogies | | | | |
| 3P&3I | 74 | 17,306 | *17,454* | *17,497* |
| 2P&3I | 71 | 17,313* | 17,455 | 17,577 |
| 3P&2I | 40 | 17,497*** | 17,577 | 17,746 |
| 2P&2I | 37 | 17,508 | 17,582 | 17,738 |
| Within persons | | | | |
| Matrices | | | | |
| 3P&3I | 111 | 13,454 | *13,676* | 14,145 |
| 2P&3I | 108 | 13,481*** | 13,697 | 14,153 |
| 3P&2I | 76 | 13,609*** | 13,762 | *14,083* |
| 2P&2I | 73 | 13,632 | 13,778 | 14,086 |
| Within items | | | | |
| Matrices | | | | |
| 3P&3I | 76 | 13,563 | *13,715* | 14,036 |
| 2P&3I | 73 | 13,570* | 13,716 | 14,024 |
| 3P&2I | 41 | 13,638** | 13,720 | 13,893 |
| 2P&2I | 38 | 13,649 | 13,725 | *13,885* |

[a] The statistical test comparing the 3P&3I model with the 2P&3I model is a mixture $\chi^2$-test, and comparing the 3P&3I model with the 3P&2I model it is a likelihood-ratio test. Because all these tests are significant, the 3P&3I model is not compared also with the 2P&2I model.
[b] The value indicating the best fitting model is indicated in italics.
* $p \leq 0.05$.
** $p \leq 0.01$.
*** $p \leq 0.001$.

the best model in only one out of the four comparisons. In the other three comparisons, the 3P&2I or the 2P&2I models seem the best. The BIC results are not surprising given the high correlations between the fast and slow abilities and the also high correspondence between the fast and slow item difficulties. The penalty for number of free parameters is higher in the BIC, so that the additional free parameters for the two dimensions do not pay off if the dimensions are highly correlated. The correlations between the latent variables are reported and discussed next.

### 3.3. Correlations and variances

The correlations between the two accuracy abilities, fast and slow, are 0.873 and 0.879 for verbal analogies and 0.880 and 0.869 for matrices, for the split within persons and within items, respectively. These correlations are estimated model parameters. Because the difficulties are modeled with fixed effects, no such correlations are available from the model estimation. In principle, one can derive correlations from the individual item difficulty estimates, but they would have a different status. Other differences between fast and slow are that the estimated variances are larger for fast than for slow. The variances are direct model estimates and not derived from drawing plausible values or other estimates available with ConQuest in a second step after the model is estimated (Wu et al., 2007). For verbal analogies the variance estimates for slow and fast are 1.19 and 2.77 with a split within persons, and 1.22 and 2.02 with a split within items. The corresponding variances for matrices are 0.85 and 1.71, and 1.02 and 1.59.

A model with two different dimensions for fast and slow includes, along with other parameters, two variances and one correlation for the latent variables, while a one-dimensional model will only include one variance. A test statistic comparing the goodness of fit between the two models will cover the two extra parameters simultaneously, while we may be interested in a finer distinction where both the equality of the variances and the magnitude of the correlation are meaningful in their own right. Therefore we have performed a targeted additional analysis, one where also the item difficulties are treated as random variables (De Boeck, 2008), so that also for the items the variances and the correlation between fast and slow can be estimated as parameters of the model. This implies that the items are treated in a similar way as the persons and that the item difficulties are considered as latent item variables. Just as $\theta_{p2}$ and $\theta_{p3}$ are latent variables, also $\beta_{i2}$ and $\beta_{i3}$ will be treated as latent variables. It is not common practice, but it fits nicely with the purpose of the additional analysis.

### 3.4. Additional analysis

In order to find out whether the variance or the correlation, or both, are at the basis of the findings, a slightly reformulated model has been estimated. There are two differences with the previous models. First, the item difficulties are treated as random variables for reasons explained in the previous paragraph. Second, instead of working with fast and slow as latent variables, one pair for the persons and one pair for the items, two new latent variables are defined: one general variable (common for fast and slow) and a specific one for fast. The formulation with

$$\begin{bmatrix} \theta_1 & 0 & 0 \\ 0 & \theta_2 & 0 \\ 0 & 0 & \theta_3 \end{bmatrix} \text{ and } \begin{bmatrix} \beta_1 & 0 & 0 \\ 0 & \beta_2 & 0 \\ 0 & 0 & \beta_3 \end{bmatrix} \text{ is replaced with the formulation } \begin{bmatrix} \theta_1 & 0 & 0 \\ 0 & \theta_g & \theta'_2 \\ 0 & \theta_g & 0 \end{bmatrix} \text{ and } \begin{bmatrix} \beta_1 & 0 & 0 \\ 0 & \beta_g & \beta'_2 \\ 0 & \beta_g & 0 \end{bmatrix}.$$

The two formulations are mathematically equivalent: $\theta_g = \theta_3$, $\theta_2 = \theta_g + \theta'_2$, and $\beta_g = \beta_3$, $\beta_2 = \beta_g + \beta'_2$. A multivariate normal distribution applies for the three random person variables, speed and the two newly defined ones, and similarly for the corresponding three random item variables. If the correlation between the general latent variable ($\theta_g$ or $\beta_g$) and the specific fast latent variable ($\theta'_2$ or $\beta'_2$) is positive, then the variance of the fast latent variable ($\theta_2$ or $\beta_2$) is larger than the variance of the slow latent variable ($\theta_3$ or $\beta_3$). When the correlation is negative and $-2\rho_{\theta g \theta' 2}\sigma_{\theta g}\sigma_{\theta' 2} > \sigma^2_{\theta' 2}$ (similarly for the difficulties), the reverse is true. When the correlation is extremely high (close to 1.00), then the fast latent variable ($\theta_2$ or $\beta_2$) has certainly a larger variance while its nature cannot be differentiated from the slow latent variable ($\theta_3$ or $\beta_3$). All this follows from the formula for the variance of the sum of two variables. Note that the correlations between $\theta_g$ and $\theta'_2$, and between $\beta_g$ and $\beta'_2$, are not correlations between fast and slow and may therefore not be compared with the correlations reported earlier (between $\theta_2$ and $\theta_3$). Model estimation was performed with the *lmer* function from the *lme4* package in *R* (Bates & Maechler, 2009), which is a flexible software tool for item response models within a generalized linear mixed model approach (De

Author's personal copy



Boeck et al., 2011; Doran, Bates, Bliese, & Dowling, 2007), one that can handle random item variables.

For the results on the abilities, we rely primarily on the within-item split, because the within-person split eliminates practically all the individual differences with respect to speed. The correlations between $\theta_g$ and $\theta'_2$ are 0.767 and 0.646 for verbal analogies and matrices, respectively.

For the results on the item difficulties, we rely primarily on the within-person split, because the within-item split eliminates practically all the item differences with respect to speed. The correlations between $\beta_g$ and $\beta'_2$ are 0.661 and 0.590 for verbal analogies and matrices, respectively. All these correlations are moderately high but far from perfect. Therefore, both aspects, a larger variance of fast in comparison with slow, and an imperfect correlation between fast and slow, seem to contribute to the better fit of the 3P&3I models. Apparently, fast and slow can be differentiated indeed, although perhaps somewhat better in the case of the difficulties. The other split, within-items for items, and within-persons for persons, confirms the results for the item difficulties, with correlations of 0.498 and 0.658 for verbal analogies and matrices, but not for the abilities, with corresponding correlations of 0.881 and 1.000. However, for these splits, the analysis is not a full analysis, as explained.

### 3.5. Speed and accuracy

The results regarding speed and accuracy are remarkable. The results to be reported refer to the within-item split for the latent person variables and to the within-person split for the latent item variables, for reasons explained earlier. For matrices, the latent speed variable $\theta_1$ is negatively correlated with $\theta_g$ ($-0.422$) and with $\theta'_2$ ($-0.965$), which is the discrepancy between fast and slow. For the latent item variables, the correlation between $\beta_1$ and $\beta_g$ and $\beta'_2$ is positive and rather high (0.630, 0.681). For verbal analogies, the latent speed variable $\theta_1$ is only slightly negatively correlated with $\theta_g$ ($-0.184$), and the correlation is positive with $\theta'_2$ (0.489). The results for the items show again high and positive correlations of $\beta_1$ with $\beta_g$ and $\beta'_2$ (0.792, 0.736).

For the items, the results are strong evidence for a positive relationship between speed and accuracy. Easy items are faster, difficult items take more time. For the persons, the results depend on the kind of test. For matrices, successful respondents are slower. Especially the relative success rate of fast responses compared to slow responses is highly negatively correlated with overall speed. Higher overall speed means relatively less success with fast responses. For verbal analogies, the relationship is different. Being relatively more successful with fast responses is even positively correlated with overall speed. The difference between matrices and verbal analogies is that matrix tasks are based exclusively on cognitive work, possibly executed on a spatial representation, whereas verbal analogies are partly based also on knowledge and knowledge does not tend to take time. One either knows or does not. It often does not help to use more time.

## 4. Discussion and conclusion

The results for the differentiation between fast and slow intelligence for the two kinds of tests, verbal analogies and matrices, and for the two operational definitions of fast and slow responses, are remarkably similar, which provides a good basis for generalization. However, because both matrices and verbal analogies are inductive tests, the generalization must be limited so far to this albeit broad category of tests. The fact that both investigated tests have a multiple-choice response format implies a further possible limitation.

Based on the results, the following conclusions can be drawn. First, fast and slow intelligence can be differentiated with respect to the processes involved, and with respect to the corresponding abilities as well. Fast and slow intelligence are rather strongly correlated, but two different sets of item difficulties seem to be required and also the abilities can be differentiated, so that they are nevertheless qualitatively different to some degree. Second, fast responses differentiate better than slow responses between persons as well as between items. The kind of differentiation is somewhat different, but stronger for fast responses than for slow responses.

These findings have consequences for the measurement of intelligence. First, a somewhat different kind of ability is measured for respondents with primarily slow responses compared to the ability that is measured for respondents with primarily fast responses. The difference is perhaps not substantial given the rather high correlation between fast and slow intelligence, but there is nevertheless an issue of equivalence that needs further investigation. Second, given the higher variance of fast intelligence compared to slow intelligence, the ability of fast respondents is measured in a more reliable way than the ability of slow respondents. Also this effect is a possible source of distortion.

The convergence between the results for the persons and for the items is neither a logically necessary result, nor is it imposed by the model. As explained in the introduction, different cognitive strategies may require the same cognitive resources, such as working memory and cognitive efficiency in the execution of the different processes. Also the model does not impose symmetry between persons and items. One can easily generate data with a strong divergence.

For a kind of task that requires more time to be solved, such as matrix tasks in comparison with verbal analogies, individual differences in speed seems to be negatively correlated with accuracy. Slow responders are better responders. A possible explanation is that one needs to take one's time for these items in order to find the correct solution, and that intuitive and impulsive responses do not pay off. Although fast responders are poorer responders, it is also true that the correctness of a fast response is a better differentiator for the ability than the correctness of a slow response.

Speed as measured here is speed in a rather self-paced condition, since the time limit was very lenient. The finding of Davison et al. (2011) with a math reasoning test is of interest regarding the meaning of speed in such a condition. These authors found that speed in a self-paced condition is positively correlated with level of performance in an experimenter-paced condition (with time pressure). This result is similar to the positive correlation we found in our analysis between overall speed and relative success rate of fast responses to verbal analogies. However, the analogous result for matrices was quite different in our study. The relation between fast accuracy and speed was in fact extremely negative. It would be interesting to have data about the same set of items in two

Author's personal copy



conditions: self-paced and with time-pressure, in order to investigate the meaning of the speed factor in self-paced conditions. Given our results, one should take into account that the meaning of speed may be different depending on the kind of test.

Such a study with two conditions would also be a way to find out how intelligence under time pressure is related to fast and slow intelligence in a self-paced condition. How are the ability and item difficulty as measured under time pressure related to fast and slow intelligence abilities and item difficulties? For example, one may hypothesize that the measures derived from time pressure conditions are correlated higher with fast response measures than with slow response measures. Another possible subject for further investigation is the differential and complimentary predictive validity of the three latent traits: fast and slow intelligence (in a self-paced condition) and intelligence under time pressure.

Apart from the substantive findings, also the approach we have used is a topic of discussion. The branching models seem an interesting tool to study item responses and to investigate the issue of fast and slow intelligence from a multidimensional perspective. They are a powerful tool in cognitive psychology in general as shown by Batchelder and Riefer (1999) and Erdfelder et al. (2009). The addition of random effects to capture individual differences is a rather recent development (Smith & Batchelder, 2008; Klauer, 2010), with the potential to bring models for cognitive psychology and cognitive processes closer to latent trait models and item response models. For an early attempt to use a multinomial processing tree approach for test data, see Garcia-Perez (1990).

A possible limitation of the approach we have followed is the discrete operational definition of fast and slow. The differentiation through a median value is rather arbitrary, since response time is of course gradual. It is therefore worthwhile to develop models that can deal with a continuous qualitative change. Although models do exist in which continuous response time and accuracy are combined, there are no such models for the study of time heterogeneity, with qualitative differences between abilities and processing depending on the response time. Only with such models would it be possible to overcome the limitation of the discrete operational definition we have used.

## Acknowledgments

We are grateful to Lutz Hornke for his help with making the data available for this study. The research reported in this paper was supported by grant GOA/05/04 of the K.U. Leuven.

## References

Akaike, H. (1974). A new look at the statistical model identification. *IEEE Transactions on Automatic Control*, *19*, 716–723.
Anastasi, A. (1976). *Psychological testing.* New York, NY: Macmillan.
Batchelder, W. H., & Riefer, D. M. (1999). Theoretical and empirical review of multinomial process tree modeling. *Psychonomic Bulletin & Review*, *6*, 57–86.
Baddeley, A. D. (1986). *Working memory.* New York, NY: Oxford University Press.
Bates, D., & Maechler, M. (2009). lme4: Linear mixed-effects models using S4 classes. http://cran.R-project.org/lme4
Bejar, I., Chaffin, R., & Embretson, S. (1991). *Cognitive and psychometric analysis of analogical problem solving.* New York, NY: Springer.
Carroll, J. B. (1993). *Human cognitive abilities.* Cambridge: Cambridge University Press.
Davison, M. L., Semmes, R., Huang, L., & Close, C. N. (2011). On the reliability and validity of a numerical reasoning speed dimension derived from response times collected in computerized testing. *Educational and Psychological Measurement*, *25*, 2011 published online May.
De Boeck, P. (2008). Random item IRT models. *Psychometrika*, *73*, 533–559.
De Boeck, P., Bakker, M., Zwitser, R., Nivard, M., Hofman, A., Tuerlinckx, F., & Partchev, I. (2011). The estimation of item response models with the lmer function from the lme4 package in R. *Journal of Statistical Software*, *39* issue 12.
Dennis, I., & Evans, J. (1996). The speed-error trade-off problem in psychometric testing. *British Journal of Psychology*, *87*, 105–129.
Doran, H., Bates, D., Bliese, P., & Dowling, M. (2007). Estimating the multilevel Rasch model: With the lme4 package. *Journal of Statistical Software*, *20* issue 2.
Erdfelder, E., Auer, T., Hilbig, B. E., Assfalg, A., Moshagen, M., & Nadarevic, L. (2009). Multinomial processing tree models: A review of the literature. *Zeitschrift für Psychologie*, *217*, 108–124.
Garcia-Perez, M. A. (1990). A comparison of two models of performance in objective tests: Finite state versus continuous distributions. *British Journal of Mathematical and Statistical Psychology*, *43*, 73–91.
Gray, J. R., Chabris, C. F., & Braver, T. S. (2003). Neural mechanisms of general fluid intelligence. *Nature Neuroscience*, *6*, 316–322.
Grudnik, J. L., & Kranzler, J. H. (2001). Meta-analysis of the relationship between intelligence and inspection time. *Intelligence*, *29*, 523–535.
Gulliksen, H. (1950). *Theory of mental tests.* New York, NY: Wiley.
Gustafsson, J. -E. (1984). A unifying model for the structure of intellectual abilities. *Intelligence*, *8*, 179–203.
Hick, W. E. (1952). On the rate of gain of information. *Quarterly Journal of Experimental Psychology*, *4*, 11–26.
Hornke, L. (1999). Benefits from computerized adaptive testing as seen in simulation studies. *European Journal of Psychological Assessment*, *15*, 91–98.
Hornke, L. F. (2001). Item generation models for higher order cognitive functions. In S. Irvine, & P. Kyllonen (Eds.), *Item generation* (pp. 159–178). Hillsdale, NJ: Erlbaum.
Hornke, L. F., & Habon, M. W. (1986). Rule-based item bank construction and evaluation within the linear logistic framework. *Applied Psychological Measurement*, *10*, 369–380.
Hornke, L. F., & Rettig, K. (1993). Evaluation und Revision einer Itembank von Analogieaufgaben [Evaluation and revision of an item bank of verbal analogy items]. *Zeitschrift für Differentielle und Diagnostische Psychologie*, *14*, 113–128.
Hornke, L. F., & Wilding, U. (1997). *Konstanz von Itemparametern bei parallelen Itembanken [Constancy of item parameters in parallel item banks] (Tech. Rep.).* : RWTH Aachen University.
Hunt, E. B. (1978). Mechanisms of verbal ability. *Psychological Review*, *85*, 109–130.
Kelley, T. (1927). *Interpretation of educational measurements.* Yonkers, NY: World Book.
Klauer, K. C. (2010). Hierarchical multinomial processing models: A latent-trait approach. *Psychometrika*, *75*, 70–98.
Kvist, A. V., & Gustafsson, J. -E. (2008). The relation between fluid intelligence and the general factor as a function of cultural background: A test of Cattell's investment theory. *Intelligence*, *36*, 422–436.
Levine, A. (1950). Construction and use of verbal analogy items. *Journal of Applied Psychology*, *34*, 105–107.
Marshalek, B., Lohman, D., & Snow, R. (1983). The complexity continuum in the radex and hierarchical models of intelligence. *Intelligence*, *7*, 107–127.
Molenberghs, G., & Verbeke, G. (2003). Likelihood ratio, score, and Wald tests in a constrained parameter space. *The American Statistician*, *61*, 1–6.
Neubauer, A. C., & Bucik, V. (1996). The mental speed–IQ relationship: Unitary or modular ? *Intelligence*, *26*, 23–48.
Partchev, I., De Boeck, P., & Steyer, R. (2011). How much power and speed is measured in this test? *Assessment* (published online June).
Rypma, B., & Prabhakaran, V. (2009). When less is more and when more is more: The mediating roles of capacity and speed in brain-behavior efficiency. *Intelligence*, *37*, 207–222.
Schwarz, G. E. (1978). Estimating the dimension of a model. *The Annals of Statistics*, *6*, 461–464.
Schweizer, K., Goldhammer, F., Rauch, W., & Moosburger, H. (2007). On the validity of Raven's Matrices Test: Does spatial ability contribute to performance? *Personality and Individual Differences*, *43*, 1998–2010.
Semmes, R., Davison, M. L., & Close, C. (2011). Modeling individual differences in numerical reasoning speed as a random effect of response time limits. *Applied Psychological Measurement*, *35*, 433–446.

Author's personal copy



Shiffrin, R. M., & Schneider, W. (1977). Controlled and automatic human information processing: II. Perceptual learning, automatic attending, and a general theory. *Psychological Review*, *84*, 127–190.
Smith, J. B., & Batchelder, W. H. (2008). Assessing individual differences in categorical data. *Psychonomic Bulletin & Review*, *15*, 713–730.
Sheppard, L. D., & Vernon, P. A. (2007). Intelligence ans speed of information processing: A review of 50 years of research. *Personality and Individual Differences*, *44*, 247–259.
Spearman, C. (1927). *The abilities of man.* New York, NY: Macmillan.
Sternberg, R. (1977). *Intelligence, information processing, and analogical reasoning: The componential analysis of human abilities.* Hillsdale; NJ: Erlbaum.
Thurstone, L. L. (1938). *Primary mental abilities.* Chicago, IL: University of Chicago Press.
Troche, S., & Rammsayer, T. (2009). Temporal and non-temporal sensory discrimination and their predictions of capacity-and speed-related aspects of psychometric intelligence. *Personality and Individual Differences*, *47*, 52–57.
Troche, S. J., & Rammsayer, T. H. (2009). The influence of temporal resolution power and working memory capacity on psychometric intelligence. *Intelligence*, *37*, 479–486.
Tutz, G. (1990). Sequential item response models with an ordered response. *British Journal of Mathematical and Statistical Psychology*, *43*, 39–55.
Ullstadius, E., Carlstedt, B., & Gustafsson, J. -E. (2008). The multidimensionality of verbal analogy items. *International Journal of Testing*, *8*, 166–179.
van Breukelen, G. J. (2005). Psychometric modelling of response speed and accuracy with mixed and conditional regression. *Psychometrika*, *70*, 359–376.
van der Linden, W. J. (2009). Conceptual issues in response-time modeling. *Journal of Educational Measurement*, *46*, 247–272.
Vernon, P. A. (1983). Speed of information processing and general intelligence. *Intelligence*, *7*, 53–70.
Vernon, P. A., & Kantor, L. (1986). Reaction time correlations with intelligence test scores obtained under either timed or untimed conditions. *Intelligence*, *10*, 315–330.
Vernon, P. A., Nador, S., & Kantor, L. (1985). Reaction time and speed-of-processing: Their relationship to timed and untimed measures of intelligence. *Intelligence*, *9*, 357–374.
Vickers, D., Nettelbeck, T., & Willson, R. J. (1972). Perceptual indices of performance: The measurement of 'inspection time' and 'noise' in the visual system. *Perception*, *1*, 263–295.
Waiter, G. D., Deary, I. J., Staff, R. T., Murray, A. D., Fox, H. C., Starr, J. M., & Whalley, L. J. (2009). Exploring possible neural mechanisms of intelligence differences using processing speed and working memory tasks: An fMRI study. *Intelligence*, *37*, 199–206.
Wang, T., & Hanson, B. A. (2005). Development and calibration of an item response model that incorporates response times. *Applied Psychological Measurement*, *29*, 323–339.
Whitely, S. E. (1976). Solving verbal analogies: Some cognitive components of intelligence test items. *Journal of Educational Psychology*, *68*, 234–242.
Wilhelm, O., & Schulze, R. (2002). The relation of speeded and unspeeded reasoning with mental speed. *Intelligence*, *30*, 537–554.
Wu, M. L., Adams, R. J., Wilson, M. R., & Haldane, S. A. (2007). *ACERConQuest Version 2: Generalized item response modeling software.* Camberwell: Australian Council for Educational Research.