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COMMENT

The Bayesian Evaluation of Categorization Models: Comment on Wills and Pothos (2012)

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Wills and Pothos (2012) reviewed approaches to evaluating formal models of categorization, raising a series of worthwhile issues, challenges, and goals. Unfortunately, in discussing these issues and proposing solutions, Wills and Pothos (2012) did not consider Bayesian methods in any detail. This means not only that their review excludes a major body of current work in the field, but also that it does not consider the body of work that provides the best current answers to the issues raised. In this comment, we argue that Bayesian methods can be—and, in most cases, already have been—applied to all the major model evaluation issues raised by Wills and Pothos (2012). In particular, Bayesian methods can address the challenges of avoiding overfitting, considering qualitative properties of data, reducing dependence on free parameters, and testing empirical breadth.

Keywords: model evaluation, Bayesian statistics, categorization models

In their review, Wills and Pothos (2012) raised a number of challenges for the evaluation of formal models in psychology, focusing on models of categorization as a case study. In particular, they argued that the evaluation of psychological models should avoid overfitting, take qualitative properties of data seriously, reduce dependence on free parameters, and test empirical breadth. We fully agree these are worthwhile goals and suspect that many others do too.

Wills and Pothos (2012) presented a series of recommendations to address these challenges. Unfortunately, their proposals fail to give serious consideration to Bayesian methods. This is a clear omission in the context of a review, since Bayesian methods have been advocated prominently as a means of evaluating psychological models in general (e.g., Kruschke, 2010, 2011; Lee, 2004; Lee & Wagenmakers, 2005; I. J. Myung & Pitt, 1997; Pitt, Myung, & Zhang, 2002; Shiffrin, Lee, Kim, & Wagenmakers, 2008; Vanpaemel, 2010) and have been applied to the evaluation of categorization models in particular (Lee, 2008; Lee & Vanpaemel, 2008; Lee & Wetzels, 2010; Vanpaemel, 2011; Vanpaemel & Lee, 2007; Vanpaemel & Storms, 2010; Voorspoels, Storms, & Vanpaemel, 2011).

More important, as we argue in this comment, it is a serious omission in the context of making good recommendations. Bayesian methods provide the principles and tools needed to address all four of the major issues discussed by Wills and Pothos (2012). In this comment, we discuss each issue in turn. For each, we explain how Bayesian methods address the problem, cite existing research in psychology in which Bayesian methods are used to address the problem, and discuss why we consider the Bayesian approach preferable to the recommendations made by Wills and Pothos (2012).

Bayesian Approaches to Four Model Evaluation Issues

Bayesian statistics is a complete, coherent, and rational approach to representing uncertainty and information, based on the axioms of probability theory (Cox, 1961; Jaynes, 2003). It is perfectly suited to the problem of drawing inferences from data using formal models, and for making inferences and decisions about the adequacy of models on the basis of data. These abilities mean that Bayesian methods can address all of the major issues raised by Wills and Pothos (2012).

The Bayesian approach has advantages on both principled and practical grounds. Some of the issues raised by Wills and Pothos (2012) are automatically solved when Bayesian methods are adopted because of their inherent properties. Other issues raised by Wills and Pothos (2012) can be solved in practice using Bayesian methods, because they allow researchers to implement richer and more general evaluations of psychological models.

Avoiding Overfitting

Wills and Pothos (2012) are rightly concerned about the issue of overfitting, which occurs when a model can describe existing data very well but does not generalize well to new or related data. Overfitting is usually caused by a model being too complicated, in the sense that it is able to describe many different patterns of data.

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It is well understood in cognitive modeling that overfitting should be avoided (e.g., Nosofsky & Zaki, 2002; Pitt & Myung, 2006). Standard quantitative criteria in which the best possible fit of a model is considered, such as the sum-squared error (SSE) measure highlighted by Wills and Pothos (2012), are highly prone to overfitting.

Bayesian methods can solve the problem. Bayesian methods for model evaluation directly address the issue of overfitting. Bayesian methods include prior and posterior predictive Bayesian assessments of model adequacy, such as the Bayes factor. The key property of these methods is that they evaluate how well a model fits data on average over a set of parameter values, rather than how well it fits data in the best-case scenario at a single parameter value. Measures of average fit automatically balance goodness of fit with model complexity. More complicated models are, in their statistical definition, those that are able to predict more data patterns (I. J. Myung, Balasubramanian, & Pitt, 2000). Since a measure of average fit includes those predictions that poorly fit the observed data, more complicated models are penalized by Bayesian methods. The founding principles of Bayesian statistics thus guarantee that they address the issue of overfitting.

Bayesian methods have solved the problem. Bayesian model evaluation methods were developed long ago in statistics (e.g., Jeffreys, 1935, 1961) and are now standard textbook material (e.g., Casella & Berger, 2002; Gelman, Carlin, Stern, & Rubin, 2004; Kass & Raftery, 1995; Robert, 1994). In psychology, Bayesian methods feature as early as Edwards, Lindman, and Savage (1963) and have been promoted heavily in the last 15 years, starting with I. J. Myung and Pitt (1997) and growing from there (e.g., Kruschke, 2010, 2011; Lee, 2004; Pitt et al., 2002; Shiffrin et al., 2008; Vanpaemel, 2010). Many articles in the cognitive modeling literature now have used Bayesian methods to address the issue of overfitting (e.g., Gallistel, 2009; Kemp & Tenenbaum, 2008; Steyvers, Lee, & Wagenmakers, 2009), including some focused on categorization models (e.g., Lee, 2008; Lee & Wetzels, 2010; Vanpaemel & Storms, 2010; Voorspoels et al., 2011).

Limitations of Wills and Pothos' (2012) proposals. Wills and Pothos (2012) briefly mentioned Bayesian model evaluation measures but did not acknowledge that such measures solve the problem of overfitting. Instead, Wills and Pothos (2012, p. 111) advocated evaluating models in terms of their ability to capture ordinal properties of data as a remedy to overfitting. A complicated model, however, can overfit ordinal data patterns as easily as it can overfit the quantitative details of the data. Thus, adopting Bayesian methods appears a better solution to the problem of overfitting than the proposal made by Wills and Pothos (2012).

Taking Qualitative Properties of Data Seriously

Wills and Pothos (2012) highlighted the importance of qualitative properties of the data, such as ordinal properties, for evaluating psychological models (see also Nosofsky & Palmeri, 1997a; Pitt, Kim, Navarro, & Myung, 2006). Although we are critical of the argument that ordinal property evaluation should be preferred to reduce the risk of overfitting, we do agree that going beyond the quantitative minutiae of the data is a worthwhile goal. In particular, it is sensible to place strong evaluative emphasis on the ability of models to describe those features of data that are the most scientifically important. These may be ordinal constraints, as focused on by Wills and Pothos (2012), or may be more general. It may be important, for example, that a model is able to capture the crossing of two learning curves, a heavy tail in a response time distribution, or a nonmonotonicity in performance. Standard quantitative criteria like SSE are not naturally sensitive to these sorts of qualitative or phenomenological features of data.

Bayesian methods can solve the problem. Emphasizing some aspects of data over others is done naturally within a Bayesian framework for evaluation. The basic approach is to overlay utilities on the probabilities produced by Bayesian inference. Augmenting probabilities with utilities affords the ability to evaluate models relative to specific properties of the data, rather than strictly in terms of their likelihood given data. So, for example, if an important aspect of evaluation is that two learning curves cross, a large utility can be placed on that property, and a model will be positively evaluated to the extent that it is inferred to satisfy that property. The advantage of imposing utilities within the Bayesian framework is that the uncertainty about model parameters is automatically and completely incorporated into the utility comparisons. Thus, while Bayesian methods do not automatically solve the problem, they provide the right tools to address the problem.

Bayesian methods have solved the problem. Incorporating utility functions is the cornerstone of Bayesian decision theory, which is textbook material in statistics (e.g., Berger, 1985; Bernado & Smith, 2000). To be fair, applications of the Bayesian decision theoretic framework are hard to find in psychology. Exceptions include J. I. Myung and Pitt (2009) and Zhang and Lee (2010), but these authors had slightly different goals than the immediate evaluation of cognitive models. More general and flexible decision criteria are desirable in many cases of model evaluation, including in categorization, and the Bayesian solution is not used as widely as it should be. One approach that is closely related to the Bayesian approach, parameter space partitioning, focuses on evaluating ordinal criteria and has been applied to the evaluation of a range of psychological models (Pitt et al., 2006; Pitt, Myung, Montenegro, & Pooley, 2008).

Limitations of Wills and Pothos' (2012) proposals. Wills and Pothos (2012) discussed examples in which primacy appeared to have been given to ordinal properties of the data but, in fact, had not. Both Nosofsky and Palmeri (1997b) and Love, Medin, and Gureckis (2004) were portrayed by Wills and Pothos (2012, pp. 111, 113, & 119) as having evaluated models based on their ordinal predictions. But, in both studies, the models were first fit quantitatively, using a measure related to SSE, and from these quantitative fits, the qualitative model behavior was assessed. Hence, these studies did not directly compare the model's ordinal predictions with the ordinal properties of the data. Thus, while their discussion reinforces the point that qualitative properties are important, it does not provide methods for incorporating them directly into formal evaluation.

Reducing Dependence on Free Parameters

Wills and Pothos (2012) were concerned about free parameters in psychological models (see also Nosofsky, 1998). We agree that minimizing the dependence of successful models on free parameters is often a worthwhile goal. Free parameters tend to reduce the empirical content or assertive power of a theory (Popper, 1959). Theories with high empirical content tell us more about the world

than theories low in assertive power. Models that quantify substantive theory are valued most when they are high in empirical content and thus depend only moderately on free parameters. For these models, free parameters correspond to meaningful psychological variables, and therefore their existence often reflects important incompleteness or immaturity in theorizing. Treating parameters as entirely free to vary, and fitting the same model parameters for multiple data sets separately, which is a standard practice, corresponds to admitting a lack of the theory needed to constrain possible values and ignores the collective empirical information that related experiments provide about parameters.[1]

Bayesian methods can solve the problem. Bayesian methods allow—in fact, require—information about parameters to be expressed formally in a model using prior distributions on model parameters. Some approaches to specifying parameters priors are designed to represent a default state of knowledge (Kass & Wasserman, 1996), whereas informative priors represent theoretical assumptions or other relevant knowledge about the psychological variables they represent. As better theories are developed and more information about psychological variables becomes available, Bayesian methods naturally incorporate this knowledge. Informative priors automatically reduce the dependence of a model on free parameters, by making the parameters less "free" and so simultaneously decrease the flexibility and increase the empirical content of the model. Thus, Bayesian methods provide the right tools to address the problem but do not automatically solve the problem, as many Bayesian model evaluations do not use informative priors but rather rely on uniform, flat, or other priors intended to be "uninformative" or "weakly informative."

Bayesian methods have solved the problem. The development of formal methods for representing information in prior distributions is a major enterprise in Bayesian statistics (Bernado & Smith, 2000; Jaynes, 2003). Maximum entropy and transformational invariance methods have been developed to give principled ways of incorporating prior information (e.g., Jaynes, 2003; Tribus & Fitts, 1968). The case for constructing psychologically meaningful priors and examples of how this can be done with psychological models are provided by Vanpaemel (2009b, 2010). Demonstrations of capturing theory in informative priors in the context of categorization models can be found in Lee and Vanpaemel (2008); Vanpaemel (2011); and Vanpaemel and Lee (2007). These authors have focused on the varying abstraction model of category learning, which assumes that people learn categories by clustering exemplars (VAM; Vanpaemel & Storms, 2008). They showed how the prior in the VAM can be used to express the theoretical assumption that exemplar clustering is likely to be driven by similarity, a theoretical position consistent with the SUSTAIN model (Love, Medin, & Gureckis, 2004) and the rational model of categorization (Anderson, 1991; Griffiths, Canini, Sanborn, & Navarro, 2007).

Limitations of Wills and Pothos' (2012) proposals. Wills and Pothos (2012, pp. 112, 113) proposed two remedies for the challenge of reducing the dependence of models on free parameters. One is to use a universal or global parameter, whose specification is general to the whole domain of phenomena that the model is intended to address. The second is to remove the free parameters by fixing each to a single value before seeing the data.

Wills and Pothos (2012, p. 113) themselves seemed to concede the first proposal is limited, as they were quick to emphasize that parameter values are meant to change across experimental conditions. Indeed, a key property of useful psychological models is selective influence, which requires that independent experimental manipulations correspond to changes in the estimated values of single model parameters. Selective influence has been evaluated in relation to a number of psychological models (e.g., Rouder et al., 2008; Voss, Rothermund, & Voss, 2004) and discussed in the context of category learning models (e.g., Vanpaemel, 2009a).

The proposal to evaluate the fit of a model at a single parameter value that is dictated by theory is a laudable goal, but seems unrealistic in practice, at least with the current state of psychological theorizing. Often, there is information about parameters available from existing data collected in related experiments or from psychological theory, but this information is unlikely to be complete enough to fix parameters to single values. Evaluation of a model with a single, unrealistically precise value often fails to incorporate important levels of uncertainty.

Wills and Pothos (2012, p. 113) discussed Nosofsky's (1984) use of theorizing about how learners allocate their attention to stimulus dimensions to fix the free attention parameters from the generalized context model (GCM). Recently, Vanpaemel and Lee (in press) demonstrated how Bayesian methods can be used to capture the attention allocation assumption in an informative prior *distribution* over parameter values rather than in a single point value. In this way, the attention parameter is neither entirely free, as in common practice, nor precisely constrained, as Wills and Pothos (2012) advocated. Instead, by using informative prior distributions in a Bayesian setting, the GCM is evaluated in a way that takes into account existing theorizing about attention, but at the same time acknowledges that this theorizing is incomplete. The Bayesian approach thus allows dependence on free parameters to be reduced, without necessitating the overconfident reduction to single values.

Testing Empirical Breadth

Wills and Pothos (2012) argued that an important aim in model evaluation is to capture data from a broad set of experiments, rather than from one or two experiments, as seems typical in practice (see also Smith & Minda, 2000). We agree that a hallmark of good theorizing and modeling throughout the empirical science is that fundamental variables and processes play roles in a wide range of experiments and phenomena. In classical physics, the underlying mass of an object affects its measured weight, acceleration, momentum, kinetic energy, and so on. In psychology, the underlying acuity of a person's memory affects that person's measured ability in recall tasks, recognition tasks, recollection tasks, and so on.

Bayesian methods can solve the problem. An important feature of Bayesian inference is that it seamlessly applies to hierarchical or multilevel, models. The key idea of the hierarchical

[1] We are not claiming that free parameters are always undesirable in psychological modeling. When psychological models are used as measurement models, for example, as in psychometric applications, estimating free parameters is useful and sensible. In these applications, psychological models allow latent psychological variables—like cognitive abilities in individuals—to be related to people's observed task behavior, and the scientific goal is to infer the free parameters, not eliminate them.

Bayesian approach is to introduce additional structure to the simple parameter-to-data relationship that characterizes most psychological models. Among the many attractive features of this approach is that it is natural to test the ability of the same model parameters and processes to account for multiple data sets from multiple experimental tasks simultaneously. This enables testing for empirical breadth. Again, the Bayesian approach, hierarchical or otherwise, does not automatically test for empirical breadth, but it does afford the possibility.

Bayesian methods have solved the problem. Hierarchical Bayesian methods were proposed long ago in statistics (e.g., Lindley & Smith, 1972) and are now standard textbook material (e.g., Gelman et al., 2004; Gelman & Hill, 2007). The general idea behind the hierarchical Bayesian approach has been advocated repeatedly in the psychological literature (e.g., Kruschke, 2010; Lee, 2006; Lee & Vanpaemel, 2008; Rouder & Lu, 2005; Shiffrin et al., 2008), and the case for hierarchical Bayesian models allowing the evaluation of empirical breadth in psychology has been made by Lee (2011). Recent psychological applications of this approach to seek breadth can, for example, be found in Pooley, Lee, and Shankle (2011), who dealt with the combination of recognition and recall memory, and in Lee and Sarnecka (2011), who focused on a developmental area closely related to categorization. These authors show how two different tasks in which children must demonstrate conceptual understanding can be modeled simultaneously in terms of common latent developmental stage variables and different process models developed for the two different empirical tasks.

Limitations of Wills and Pothos' (2012) proposals. Wills and Pothos (2012, p. 119) proposed the evaluation of models against a large set of key phenomena in a way such that parameters are held constant across the whole set of phenomena or are fixed to single values without recourse to data. As we have discussed, in the context of reducing dependence on free parameters, both of these proposals are limited. For models with free parameters, it is often desirable that they meaningfully vary across conditions, and it seems generally the case that theory is insufficiently well developed to fix parameters to single point values.

Conclusion

The Bayesian framework provides a complete and coherent solution to the basic scientific challenge of drawing inferences over structured models from sparse and noisy data. That is exactly what is needed to evaluate psychological models, including models of categorization. In this comment, we have discussed how Bayesian methods can address the four main evaluative issues raised by Wills and Pothos (2012). We have pointed to research in which Bayesian methods have already been adopted successfully and indicated how the evaluation of psychological models will benefit even further from an expanded use of the Bayesian framework.

For one of the issues raised by Wills and Pothos (2012), the basic principles of Bayesian methods provide an automatic solution. The correct application of Bayesian inference automatically controls for model complexity and prevents overfitting. But, for the other issues, the Bayesian approach does not in and of itself provide a complete solution. Instead, it provides an appropriate framework for workable and principled solutions. Bayesian inference can operate naturally with decision theoretic utilities that can be used to emphasize the scientifically important qualitative properties of data. Bayesian inference requires the specification of a prior distribution over model parameters, and so can allow theoretical information about their content and constraints to be incorporated into evaluation. And Bayesian inference can work effectively with hierarchical models that can be applied to capture the way in which the same psychological variables and processes are manifest in multiple psychological phenomena, and so it allows for direct evaluations of empirical breadth.

Of course, it will take some effort to realize all of the potential of the Bayesian approach, and there are mundane practical considerations. Some categorization models are computationally easier to implement within a fully Bayesian analysis than others. Foundational models like the GCM are relatively straightforward, but learning models like ALCOVE (Kruschke, 1992) or SUSTAIN are more challenging. These are computational rather than conceptual issues, and the successful Bayesian evaluation of closely related learning models, like the expectancy valence model (Wetzels, Vandekerckhove, Tuerlinckx, & Wagenmakers, 2010), strongly suggests that they are surmountable.

Thus, we believe that the work required to provide an understanding of categorization and other cognitive activities is to develop good theories and models and to collect the strong empirical evidence needed to evaluate those theories and models. The challenge is not to find a framework for evaluation. The Bayesian framework already exists and is ready to do that part of the work.

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